Estimation of MIXED TRAFFIC DENSITIES in Congested Roads Using Monte Carlo Analysis

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Vehicle congestion is a serious problem throughout the world that impacts transport, health, and communication infrastructure. Estimating the number and type of vehicles in a traffic jam can greatly assist planners and modelers interested in specific areas of emission impacts and evaluating secondary congestion outcomes, such as lost time/productivity, cellular bandwidth requirements, and emergency response to large accidents.

A Novel Approach

The authors of this article developed a novel stochastic Monte Carlo approach to estimate the number and type of vehicles on congested road sections. This method assumes that each vehicle occupies road space based on its length and inter-vehicle gap during congested traffic. The inter-vehicle gap is subject to variability due to driver behavior and average speed. By assigning vehicle spaces on a road based on the average speed, each vehicle can be treated as an independent variable using Monte Carlo simulation to identify ranges of possible outcomes. Under multiple sampling, the most likely number of mixed vehicles in a 1-km unit road length can be represented by the mode or median of the distribution of results, which is normal due to the Central Limit Theorem. Once the modes of different speeds are calculated, vehicle density curves can be estimated for combined traffic and individual vehicle types.

Various models have been used to estimate traffic on roads, including statistical models, Kalman filters, and neural networks. These models looked at how traffic flowed over time to assess traffic management strategies and required complex computations and historical data to calibrate.
the necessary equations for a specific stretch of road. Monte Carlo methods have been used to validate results of traffic flow models, but not to generate results. These models look at traffic flow under various conditions and not at the extreme condition of grid lock traffic. Under this state, a simpler model can be used.

Assumptions
Vehicles in congested roads move at homogenous speeds, due to the lack of options available to individual drivers. Assuming that all vehicles follow an average speed in slow moving, grid-locked traffic is fundamental to this model. Fast moving traffic (i.e., > 40 km per hour, or KPH) is assumed to be free flowing and not applicable to this model.

The model accepts different road sections with different mixes of vehicles. For example, a highway near residential areas would have more sedans and sport utility vehicles (SUVs), while a road near a port or industrial zone would have more heavy goods vehicles and multi-axle rigs.

Methodology
The number of vehicles on a given unit road length depends on the length of the vehicle (L) and the “space cushion” a driver keeps from the car in front, or the inter-vehicle gap (IVG). The recommended IVG is around 2–3 seconds at the vehicle speed. For example, at 120 KPH, this
represents 67 m, while at 5 KPH, this represents 2.8 m. For an SUV with a length of 5 m traveling at 5 KPH, the most likely road space (RS) required to operate the vehicle is \( L + IVG = 7.8 \) m, as shown in Figure 1. Figure 2 shows the difference between vehicles spacing at different speeds assuming a 2-sec IVG, at 5 KPH and 40 KPH. The total number of vehicles (\( n \)) on 1-km road moving at the same speed can be estimated by summing the number of individual vehicle lengths (\( L_i \)) and individual IVG (IVGi):

\[
\sum_{i} n = \frac{RS_i}{H} < 1000 \text{ meters}
\]

Both IVG and L are independent variables subject to a wide range of values. A vehicle’s length may average from 1.8 m for a sedan, and up to

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Vehicle Type</th>
<th>Company</th>
<th>Model</th>
<th>Year</th>
<th>Most Likely Length (m)</th>
<th>Gross Vehicle Mass (kg)</th>
<th>Fuel Type</th>
<th>Probability (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sedan</td>
<td>Honda</td>
<td>Civic LX</td>
<td>2013</td>
<td>1.79</td>
<td>1,650</td>
<td>Petrol</td>
<td>0.55</td>
</tr>
<tr>
<td>2</td>
<td>SUV</td>
<td>Toyota</td>
<td>Prado VX</td>
<td>2013</td>
<td>4.95</td>
<td>2,990</td>
<td>Petrol</td>
<td>0.33</td>
</tr>
<tr>
<td>3</td>
<td>Bus, Midsize</td>
<td>Toyota</td>
<td>Coaster</td>
<td>2013</td>
<td>6.25</td>
<td>5,180</td>
<td>Diesel</td>
<td>0.07</td>
</tr>
<tr>
<td>4</td>
<td>Bus, Large</td>
<td>Tata</td>
<td>Starbus 54</td>
<td>2013</td>
<td>9.71</td>
<td>14,860</td>
<td>Diesel</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 3. Possible SUV road space at 5 KPH. Figure 4. Probability distribution of observed vehicles. Figure 5. Probability distribution of IVG timing. Figure 6. Total number of vehicles estimated on 1-km road at 5 KPH.
IVGs are independent of the vehicle due to driver behavior and changes in speed due to the vehicle traveling ahead. At 5 KPH, IVGs range from 0.5 to 4 m. The range of possible road space used by a 5-m long SUV may vary, as shown in Figure 3. This is especially apparent at lower speeds (i.e., < 20 KPH) due to stop-and-go driving patterns.

Specific road use is important when estimating the types of vehicles in a sample population of vehicles. During model development, four classes of vehicles were used. Vehicle classes were selected to represent existing traffic based on observations in our case study city, Kuwait City, as shown in Table 1.

Initial probabilities of occurrence (P) were assigned to a discrete probability distribution, as shown in Figure 4, such that the total probability to select a vehicle was 1. The number and type of vehicle classes can be expanded to account for better classification, such as engine size, weight, fuel types, and age.

A discrete algorithm was set up in a spreadsheet for multiple speeds ranging from 5 to 40 KPH. Table 2 shows speed sets and conversion to meters.

<table>
<thead>
<tr>
<th>KPH</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/sec</td>
<td>1.4</td>
<td>2.8</td>
<td>4.2</td>
<td>5.6</td>
<td>11.2</td>
</tr>
</tbody>
</table>
We modeled speeds less than 5 KPH as 5 KPH due to the IVG maintained by drivers at lower speeds.

We assigned vehicle spaces based on a maximum number of 401 vehicles possible on a 1-km road moving at 5 KPH. This maximum value assumes that only sedans are on the road driving at the smallest possible safe distance. During modeling, however, the number of vehicles in the same stretch of road never exceeded 170. Safe IVG timing values were assumed to range from 0.5 seconds to 2 seconds and 4 seconds (maximum) in a continuous triangle distribution, as shown in Figure 5.

Each vehicle length was assigned its own probability distribution using a pert distribution and vehicle manufacturer data. We ran our stochastic model using 5,000 iterations on each variable. During each iteration, a vehicle class was randomly selected from the 4 classes for each space. The vehicle length was then selected based on the class of vehicle. The safe distance was added to the vehicle length by randomly selecting a time spacing and multiplying it by the average speed to get the safe distance. If the cumulative length was less than 1,000 m, the class was assigned a value of 1 to allow tallying and grouping. Vehicle classes at the end of the list that exceeded the 1-km length were assigned a zero and not counted. Table 3 shows a portion of an iteration at 5 KPH.

### Results

Palisade Software’s @RISK Version 6.2 Industrial Edition was used to provide the Monte Carlo analysis using a Latin Hypercube sample generator. A total of 5,000 iterations were run at 5, 10, 15, 20, and 40 KPH at the same time. Our model captures the total number and class of vehicles in 1 km of road. Lane changing was not considered. Figure 6 shows the results of all vehicles traveling at an average of 5 KPH, including 90% confidence intervals. The expected number, and average, of total vehicles is 155 with a standard deviation of 4 vehicles. The calculated mean of different types of vehicles at different speeds are shown in Figure 7. Figure 8 shows the distribution of sedans traveling at 5 KPH.

Graphing the statistical mean for the total number of vehicles over different average speeds yields a power curve, as shown in Figure 9. Fitting the curve with a power series trend line provides very high correlation (R²) that can approximate the expected value at each speed. Similar curves (mean of each speed) for each vehicle class were prepared and summarized in Table 4. A curve for large buses was not included because the expected value at each speed is 1. Table 4 summarizes the expected number for vehicles by class in 1 km. This is the integer value of the equation, where x equals the average traffic speed in KPH. Figure 10 shows a power curve for sedans.

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Expected Number of Vehicles in 1-km Road</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedan</td>
<td># of Sedans = Integer (324.32x^-0.703)</td>
</tr>
<tr>
<td>SUV</td>
<td># of SUVs = Integer (179.11x^-0.736)</td>
</tr>
<tr>
<td>Bus, Medium</td>
<td># of Medium Buses = Integer (12.55x^-0.660)</td>
</tr>
<tr>
<td>Bus, Large</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. Sample of an iteration showing vehicle class and road space selection.

Table 4. Expected number of vehicles by class in 1-km for profile.
The results in Table 4 are representative only of that section of road with the traffic profile in Table 2. Different traffic profiles will have different densities and results. Other factors that could affect the traffic profiles include location of road section, type of road, season, time of day, weather conditions, and road construction activities.

Future Investigations
We developed a stochastic procedure to estimate the number of vehicles by class in a 1-km stretch of road at various speeds. This procedure can be linked to vehicle emission data to estimate real-time mobile source pollution on sections of road due to traffic conditions (i.e., congested vs. free flowing). Additional Monte Carlo analysis can be used to estimate the emissions from individual vehicles given expected vehicle emission rates. Validation of this procedure includes counting vehicles during various traffic conditions and identifying vehicle classes. This will require capturing a section of road and identifying individual vehicle types and separation at various speeds to reduce IVG uncertainty.

References